Exercise 69

Suppose f is differentiable on $\mathbb{R}$. Let $F(x) = f(e^x)$ and $G(x) = e^{f(x)}$. Find expressions for (a) $F'(x)$ and (b) $G'(x)$.

Solution

Take the derivative of $F(x)$.

$$F'(x) = \frac{d}{dx}[f(e^x)]$$

$$= f'(e^x) \cdot \frac{d}{dx}(e^x)$$

$$= f'(e^x) \cdot (e^x)$$

Take the derivative of $G(x)$.

$$G'(x) = \frac{d}{dx}[e^{f(x)}]$$

$$= e^{f(x)} \cdot \frac{d}{dx}[f(x)]$$

$$= e^{f(x)} \cdot f'(x)$$